Toschi et al. Reply: In the preceding Comment, Phillips and Gordon [1] suggest to subtract a phonon contribution from the specific heat c_V of LiV₂O₄ [2], which—with and without this subtraction—shows a rapid change of slope (i.e., a kink) in agreement with our theoretical prediction [3].

The proposed subtraction procedure is highly problematic since the phononic specific heat of LiV_2O_4 is not experimentally available. That of LiTi_2O_4 used for the subtraction in [1] is actually not even the experimentally measured one but it is an extrapolation to the relevant temperature range since the compound becomes superconducting below 12 K [4]. Moreover, only data from polycrystalline samples are available for LiTi_2O_4 , which in the case of LiV_2O_4 differ by up to 16% from single-crystal results [5].

However, this problematic data processing is not relevant for our conclusions since a rapid change of slope, i.e., a kink, cannot be affected by the subtraction of a smooth (phononic) T^3 term. Such subtraction can only change the range of temperatures at which first corrections to a second linear regime become visible; only in this respect do different phonon subtractions differ. As is clear from Fig. 1, the phonon contribution, in addition to being smooth, is also essentially negligible (it contributes less than 1%) at the kink temperature $T^* = 6-7$ K.

We feel it is also necessary to correct two statements of the Comment [1]. First, Phillips and Gordon [1] suggest that we would cite the linear behavior between 9 and 20 K as evidence for the general validity of our theory. This is wrong. Explicitly, we take the kink at T^* as evidence (see Fig. 3 of [3]). On the contrary, we emphasize that the analytical fit formula Eq. (3) of Ref. [3] is only valid up to about twice the kink temperature (see the derivation of [3,6] and footnote 10 of [3]). For larger temperatures, the numerical solution [3,6] shows a negative curvature which is actually in agreement with the subtracted specific heat of [1].

Second, Phillips and Gordon [1] claim that without a second linear behavior over an extended temperature interval (here up to about 20 K) the identification of a kink would be arbitrary. This is wrong. A rapid change of slope (kink) is defined by the behavior around the kink temperature T^* . What happens at temperatures far away from the T^* is irrelevant; only here the suggested phonon subtraction of [1] contributes significantly. It might be more difficult to identify the kink by the naked eye; for the mathematical definition or a computational fit, the subtraction of a smooth phonon contribution as suggested in [1] does not matter.

By hands of LiV₂O₄ data, this can be shown explicitly. The standard polynomial fit to the phonon subtracted data leads to an artificially large T^3 term for the electronic specific heat, 20 times larger than the estimated phononic T^3 term. In contrast, our analytical fit formula with basically the same parameters as in [3] still works well (see



FIG. 1 (color online). Specific heat of LiV_2O_4 versus temperature. Shown are the single-crystal raw data [2], the processed data of Johnston [7] subtracting (extrapolated) polycrystalline LiTi_2O_4 from polycrystalline LiV_2O_4 , and the processed data of the Comment [1] subtracting (extrapolated) polycrystalline LiTi_2O_4 from single crystalline LiV_2O_4 . All data sets clearly show a kink in the specific heat of LiV_2O_4 below 10 K. Also, the processed data of [1] agree well with our theoretical fit formula in the relevant temperature range, i.e., around the kink. At higher temperatures, deviations are to be expected [3,6].

Fig. 1), as expected mathematically. Only the temperature range where deviations from the analytical fit are visible is lower (and actually closer to what is to be expected [3,6]).

In conclusion, the Comment [1] is beside the point of our Letter [3], i.e., the existence of a kink in the specific heat.

A. Toschi, M. Capone, C. Castellani, and K. Held Vienna University of Technology, 1040 Vienna, Austria

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- [2] C. Urano et al., Phys. Rev. Lett. 85, 1052 (2000).
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- [4] Subtracting the phononic contribution to the specific heat of $Li_{4/3}Ti_{5/3}O_4$ (i.e., an insulator, as in the accepted standard procedure) gives an estimated phonon contribution which differs up to more than 20% from that of $LiTi_2O_4$. While $Li_{4/3}Ti_{5/3}O_4$ is most likely also not a proper reference system, it nonetheless shows the big uncertainty in the phononic specific heat contribution estimated so far. A better experimental reference system is certainly needed.
- [5] Note: the percentwise difference in c_V/T of poly- versus single-crystalline data is, in fact, not approximately constant as claimed in [1]. It ranges from more than 16% to 8%.
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