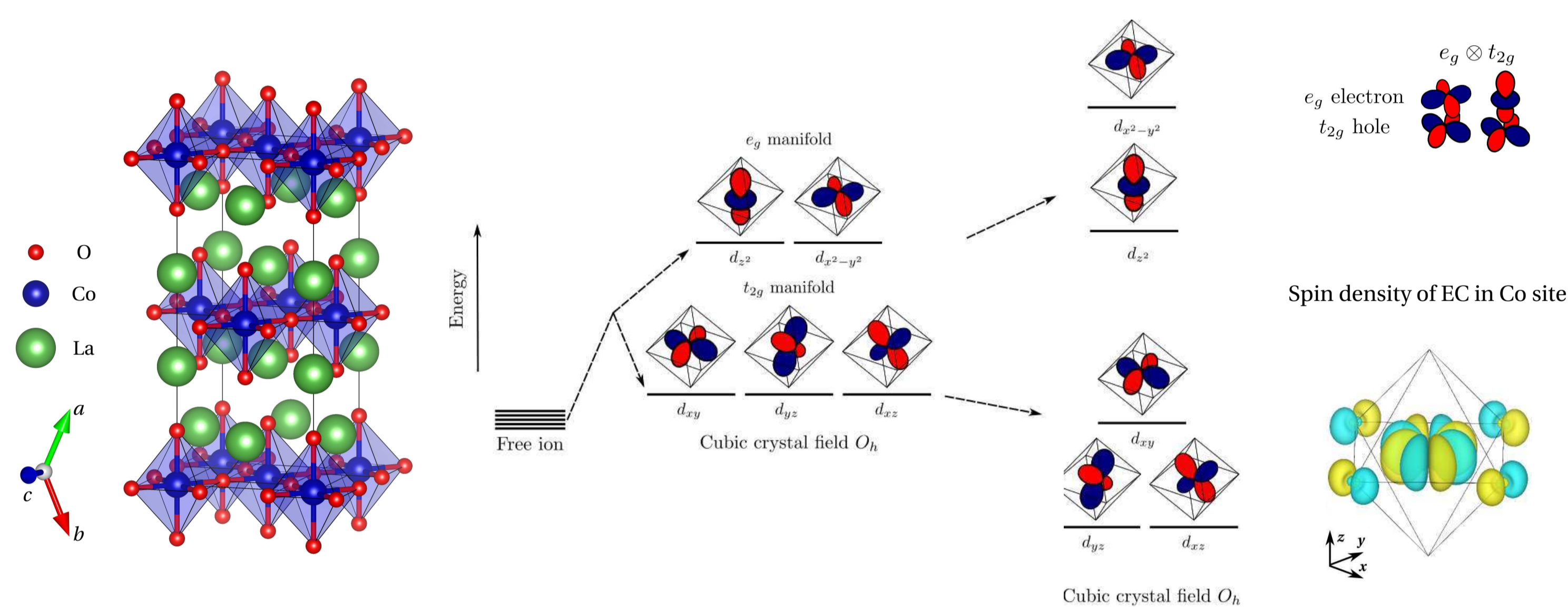


In this work we use the LDA+U approach to search for excitonic ordered ground states of  $\text{LaSrCoO}_4$ . We find an ordered state characterized by the staggered arrangement of magnetic multipoles to be stable over a broad range of Co  $3d$  interaction parameters. The ordered state can be described as a  $d_{xy} \otimes d_{x^2-y^2}$  excitonic condensate carrying spin  $S = 1$ . We have also constructed an effective strong-coupling model in to investigate the excitonic gap. We also provide a comparison with previous studies on  $\text{LaCoO}_3$  and we find that for the same interaction parameters the excitonic gap is smaller (possibly vanishing) in the layered cobaltite.

## 1 DFT computational details

The  $\text{La}_{2-x}\text{Sr}_x\text{CoO}_4$  structure consists of single layers of  $\text{CoO}_6$  corner-sharing octahedra separated by a random distribution of La and Sr ions. In order to investigate the possibility of excitonic condensation we simulate the  $\text{LaSrCoO}_4$  compound employing the virtual crystal approximation on the La atom ( $Z_{\text{La}} = 56.5$ ). With this approach the Co will have +3 valence with a  $3d^6$  electronic configuration. The unit cell vectors used in the calculation are highlighted in on the picture below left panel and correspond to the parameters  $a = 6.8019 \text{ \AA}$ ,  $b = 6.8019 \text{ \AA}$ ,  $c = 5.3796 \text{ \AA}$ ,  $\alpha = \pi/2$ ,  $\beta = \pi/2$ , and  $\gamma = 2.3284$ .

The calculations were performed within the density functional theory framework. We used the LDA as exchange-correlation potential and the Coulomb interaction 'U' added as a spin-polarized mean field term for the  $3d$  shell of Co. The double-counting was corrected with the so called fully localized limit. The Brillouin zone was sampled with a  $4 \times 4 \times 4$  k-mesh, the muffin-tin radii was set to: 2.5 for La, 1.91 for Co and 1.65 for O in bohr units, and the plane wave cut-off  $R_{\text{mt}}K_{\text{max}} = 6$ . The calculations were carried out with the WIEN2k software package.



## 2 Excitonic order parameter

In compounds with a cubic crystal field and  $d^6$  electronic filling, a low-energy exciton is a bound state of an  $e_g$  electron and a  $t_{2g}$  hole. These excitons correspond to excitations from low spin  $LS$  to intermediate spin states  $IS$ . Due to the layered structure of  $\text{La}_2\text{CoO}_4$  only the excitons with  $d_{x^2-y^2} \otimes d_{xy}$  orbital configuration will condense. The excitonic order will contribute with an extra anomalous term to the density matrix

$$\Gamma' = 4i \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \quad \Gamma'' = 4i \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad D = D_0 + \sum_{\beta=x,y,z} (\phi'_\beta \Gamma' + \phi''_\beta \Gamma'') \otimes \tau_\beta,$$

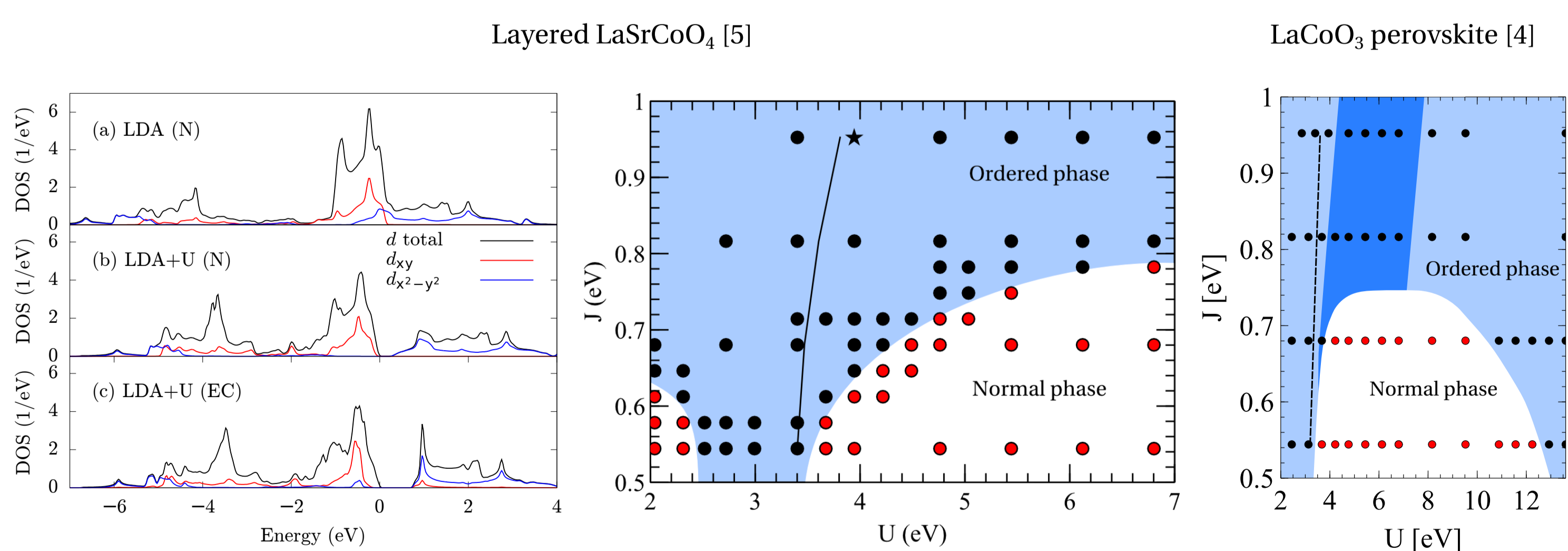
where  $\Gamma'$  and  $\Gamma''$  are expressed in the spherical harmonic basis and  $D_0$  is the occupation matrix in the normal phase. We can describe this excitonic state with a three-dimensional complex order parameter  $\phi$ .

$$\hat{O}'_\beta = \frac{i}{4} \sum_{\sigma, \sigma'=\uparrow, \downarrow} (\tau_\beta)_{\sigma\sigma'} (\hat{c}_{2\sigma}^\dagger \hat{c}_{-2\sigma'} - \hat{c}_{-2\sigma}^\dagger \hat{c}_{2\sigma'}) \\ \hat{O}''_\beta = \frac{i}{4} \sum_{\sigma, \sigma'=\uparrow, \downarrow} (\tau_\beta)_{\sigma\sigma'} (\hat{c}_{-2\sigma}^\dagger \hat{c}_{-2\sigma'} - \hat{c}_{2\sigma}^\dagger \hat{c}_{2\sigma'}) \quad \phi = \langle \hat{O}'_\beta \rangle + i \langle \hat{O}''_\beta \rangle = \phi'_\beta + i \phi''_\beta,$$

where  $\hat{c}_{m\sigma}$  ( $\hat{c}_{m\sigma}^\dagger$ ) are the creation (annihilation) operators for  $3d$  electrons on the same Co atom (the site indices are not shown for sake of simplicity). The orbital index  $m$  refers to the spherical harmonic  $Y_{2,m}$  basis with the  $z$  quantization axis. The Pauli matrices  $\tau_\beta$  ( $\beta = x, y, z$ ) capture the spin-triplet character of the exciton. When the order parameter is zero, there is no excitonic order and the excitonic spin density on the Co site vanishes.

### 2.1 LDA+U calculations

LDA+U calculations without SOC lead to a non-zero excitonic order parameter  $\phi = (-1)^k(0, 0, \lambda')$  with staggered arrangement in the Co-O plane. This electron-hole coupling lowers the energy and increases the band gap. We studied the stability of the EC as function of the Coulomb interaction  $U$  and the Hund's coupling  $J$ . The phase diagram resembles previous results obtained for  $\text{LaCoO}_3$ .



The spin-orbit interaction introduces a non-zero value of the hybridization between  $e_g$  and  $t_{2g}$  orbitals.  $\langle xy, \sigma | \hat{l}_z \hat{s}_z | x^2 - y^2, \sigma \rangle$  coupling generates a non-zero contribution to  $\phi''_z$ . For  $U = 3.94$  and  $J = 0.95$  eV (star-shaped point in the  $\text{La}_2\text{CoO}_4$  phase diagram) we found two distinct excitonic phases with different symmetries. As shown in the table, the excitonic in-plane excitonic order was found to be the ground state.

Staggered	Uniform	Solution	Approach	$ \phi' $	$ \phi'' $	$\Delta E$ (meV/f.u.)
		EC	LDA+U	0.289	0.000	-14.25
		EC $^\perp$	LDA+U+SOC	0.274	0.098	-14.83
		EC $^\parallel$	LDA+U+SOC	0.299	0.096	-17.58

## 2.2 Strong coupling model

We have build an effective bosonic model to study the excitonic dispersion for different interaction parameters where the starting point was the Co  $d$ -shell Hubbard hamiltonian. To this end, we used the set of the 25 lowest energy many-body LS, IS and HS sates of  $d^6$  electronic configuration to treat the non local terms of  $\hat{\mathcal{H}}_t^{(r)}$ .

Co  $d$ -shell Hubbard model

$d^6$  LS, IS, HS effective model

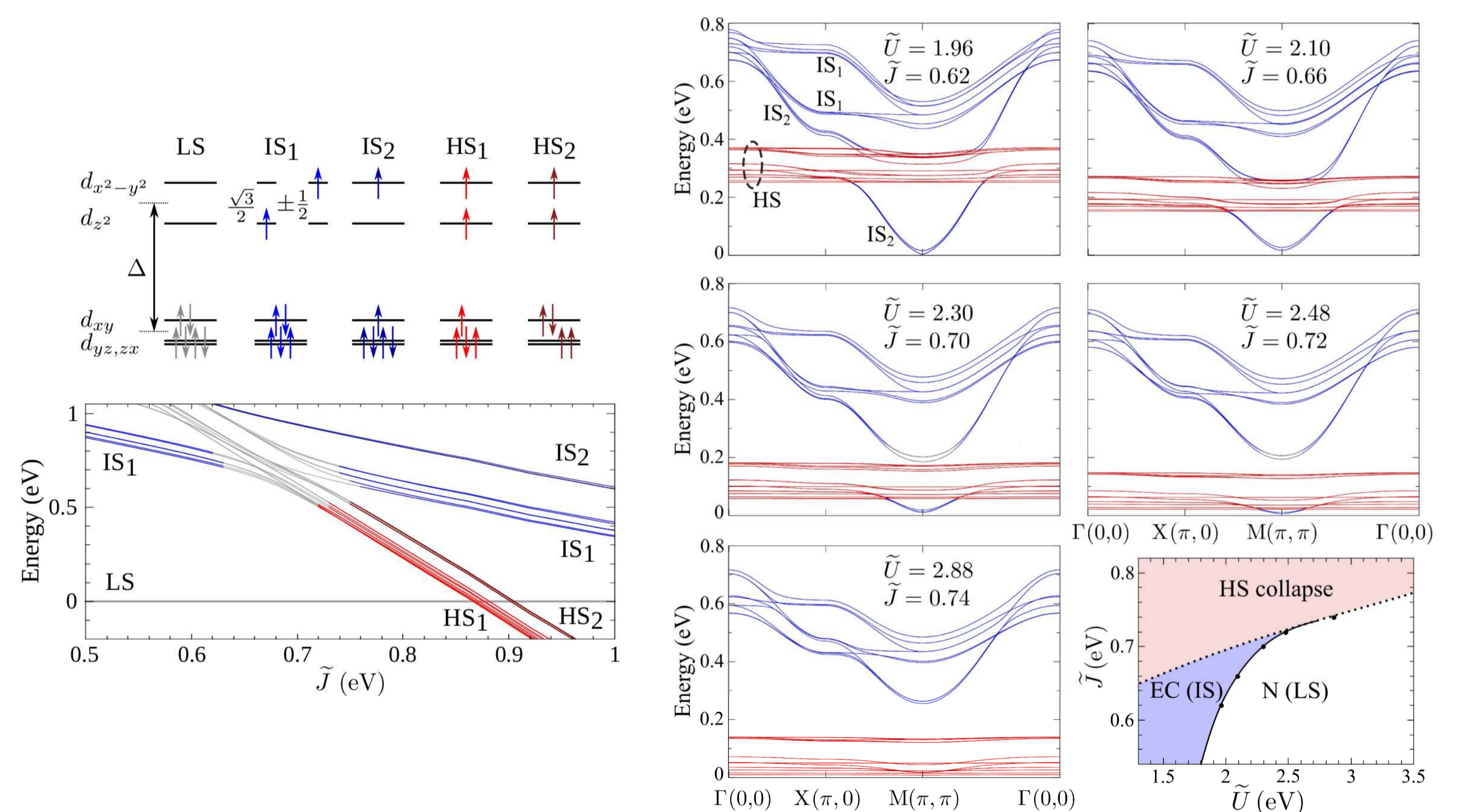
$$\hat{\mathcal{H}} = \sum_i \hat{\mathcal{H}}_{\text{at}}^{(i)} + \sum_r \hat{\mathcal{H}}_t^{(r)}$$

$$\hat{\mathcal{H}}_{\text{at}}^{(i)} = \sum_{\alpha\beta} h_{\alpha\beta}^{ii} \hat{c}_{i\alpha}^\dagger \hat{c}_{i\beta} + \sum_{\alpha\beta\gamma\delta} U_{\alpha\beta\gamma\delta} \hat{c}_{i\alpha}^\dagger \hat{c}_{i\beta}^\dagger \hat{c}_{i\gamma} \hat{c}_{i\delta}$$

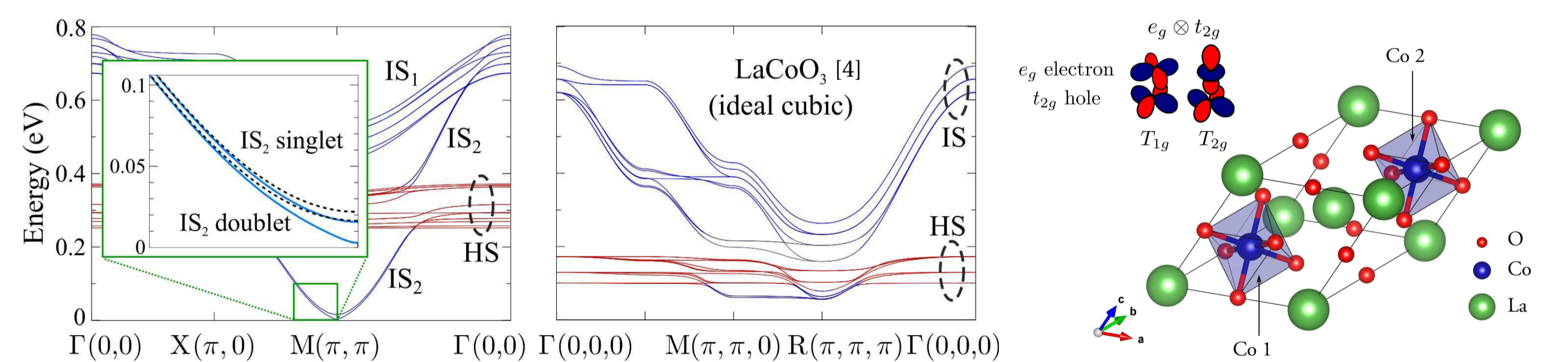
$$\hat{\mathcal{H}}_{\text{eff}} = \sum_{ij, \alpha\beta} (\epsilon_{1\alpha\beta}^{ij} \hat{d}_{i\alpha}^\dagger \hat{d}_{j\beta} \hat{s}_i \hat{s}_j^\dagger + \epsilon_{2\alpha\beta}^{ij} \hat{d}_{i\alpha}^\dagger \hat{d}_{j\beta} \hat{s}_i \hat{s}_j) + \text{H.c.} + \hat{\mathcal{H}}_{\text{int}}$$

$$\hat{\mathcal{H}}_t^{(r)} = \hat{\mathcal{H}}_t^{(ij)} = \sum_{\alpha\beta} h_{\alpha\beta}^{ij} \hat{c}_{i\alpha}^\dagger \hat{c}_{j\beta}, \quad i \neq j$$

In this model we consider the LS state as the bosonic vacuum,  $|\emptyset\rangle_i = \hat{s}_i^\dagger |0\rangle$ , and other states bosonic flavors  $\alpha$  are characterized by the creation (annihilation) operators  $\hat{d}_{i\alpha}^\dagger$  ( $\hat{d}_{i\alpha}$ ) on the lattice site  $i$ . From the on-site terms of the Hubbard model we obtained the atomic picture of the Tanabe-Sugano diagram where we set up the LS state to be the zero energy point for each  $J$  value. From the diagonalization of the effective bosonic hopping we obtained the energy dispersions of the considered LS, IS and HS manifolds. We found three distinct regions in the phase diagram: normal, IS and HS phases.



We also performed calculations on the ideal  $\text{LaCoO}_3$  perovskite to compare the excitonic gap obtained for  $\text{LaSrCoO}_4$  for  $\tilde{U} = 1.96$  and  $\tilde{J} = 0.92$  eV. We observed that the gap almost vanishes in layered  $\text{LaSrCoO}_4$  cobaltite while it is sizable (around 55 meV) for  $\text{LaCoO}_3$  compound. Also notice that the HS excitations lay higher for the layered structure than for the cubic one.



## 3 Conclusions

- LDA+U calculations showed that the excitonic condensate is a stable solution for a large range of  $U$  and  $J$  interaction parameters studied.
- This excitonic phase was found to lower the energy of the non-ordered state.
- The effective strong-coupling model suggest that the  $\text{LaSrCoO}_4$  is close to the excitonic instability.

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